Exercise 57

Show that the curve $y = 2e^x + 3x + 5x^3$ has no tangent line with slope 2.

Solution

Take the derivative of the given function and show that it cannot equal 2 for any value of x.

$$\begin{aligned}
y' &= \frac{d}{dx}(2e^x + 3x + 5x^3) \\
&= \frac{d}{dx}(2e^x) + \frac{d}{dx}(3x) + \frac{d}{dx}(5x^3) \\
&= 2\frac{d}{dx}(e^x) + 3\frac{d}{dx}(x) + 5\frac{d}{dx}(x^3) \\
&= 2(e^x) + 3(1) + 5(3x^2) \\
&= 2e^x + 3 + 15x^2
\end{aligned}$$

Set this equal to 2 and solve for x.

$$2e^x + 3 + 15x^2 = 2$$
$$2e^x + 15x^2 = -1$$

The left side is strictly positive because $e^x > 0$ and $x^2 \geq 0$, so it can never be equal to -1 for any value of x. Therefore, the curve $y = 2e^x + 3x + 5x^3$ has no tangent line with slope 2.